Set estimation under dependence

Laurence Reboul
Institut de Mathématiques de Marseille

(Joint work with C. Aaron and P. Doukhan)

Ecodep Conference - september 30 2024

Problem

Let $\mathbf{X} = (X_t)_{t\in\mathbb{Z}}$ be a strictly stationary process with marginal density $f : \mathbb{R}^d \to \mathbb{R}$ and nonempty compact support

$$S = \overline{\{x, f(x) > 0\}}.$$

- To estimate S and particular characteristics of the set such as its boundary, its volume and its perimeter, based on a n-sample $\mathbb{X}_n = (X_1, \dots, X_n) \subset \mathbb{R}^d$ of $\mathbf{X}$.
- Applications to ecology : estimating the home-range of species based on the observation of the spatial distribution of species or the movement of one or more representative members is a central problem extensively studied in the literature (Getz & al., 2004)

  $\Longrightarrow X_t \subset \mathbb{R}^d$ is the position of a member of a spieces. The home-range corresponds to S.

State of the art for i.i.d. samples

Support estimation (and related quantities) is a well studied problem when $\mathbb{X}_n$ is an i.i.d. sample.

- Devroye-Wise historical estimator (Cuevas & Rodríguez-Casal, 2004, Devroye & Wise, 1980). Built by means of a smoothed version of the sample ; no shape assumptions made on S ; has universal properties but non-optimal rates.

  $\Longrightarrow$ Shape restrictions must be imposed on S to obtain efficient solutions.

- Convex hull of the sample (Dümbgen and Walther, 1996) - achieves optimal rates when S is convex.

  $\Longrightarrow$ Convexity is too restrictive.

- **R-convex hull** (Aaron & al., 2022, Arias-Castro & al., 2016, Rodríguez-Casal, 2007, Rodríguez-Casal & al., 2022, Walther, 1997). Allows to handle a larger family of sets; nearly optimal for some sufficiently smooth sets.

- Other estimators of S have also been proposed (Aaron & al., 2016, Hardle & al., 1996) exploiting other shape restrictions (local convex hull, α-shape, positive reach, ...)

Generalizations to dependent settings

- In real life applications, the independence of the sample is generally unrealistic. This is the case for instance if a member is observed *via GPS* or if the sample consists of the trajectory of one or more members observed over a given time period.

- Set estimation under dependence has been little studied.
  - Some works (Cholaquidis & al., 2014, 2023) consider continuous time trajectories of a reflected Brownian motion.
  - A recent work (Kallel & Louhichi, 2023) consider the reconstruction of density supports of dependent stationary $\mathbb{R}^d$-valued random processes with the use of the Devroye-Wise estimator.

- Aim of the study: To extend estimation results (obtain explicit convergence rates) on the R-convex hull of the points of an i.i.d. sample to a dependent setting.

  $\Longrightarrow$ $\mathbb{X}_n$ is a $n-$sample from a strictly stationary process $\mathbf{X}$ with some kind of weak dependence condition - $\mathbf{X}$ strongly mixing or θ-weakly dependent (incidently, we also consider negative association and m-dependence).

R-convex set and R-convex hull

Definition (R-convex hull)

The R-convex hull of a set S is defined as the intersection of the complement of all open balls of radius R not meeting S:

$$C_R(S) = \left(\bigcup_{\mathring{B}(x,R)\cap S=\emptyset} \mathring{B}(x,R) \right)^c,$$

where A^c and $\mathring{A}$ denote the complementary and interior of A.

Definition (R-convex set)

$S \subset \mathbb{R}^d$ *is R-convex* $\Longleftrightarrow S = C_R(S)$

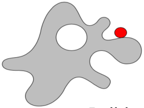


Fig. 1: R-convex set. Ball has radius R

Properties

- *A compact set S is R-convex if for every $x \in S^c$, there exists an open ball $B(y,R) \subset S^c$ such that $x \in B(y,r)$.*
- *Convex sets are R-convex, for any $R > 0$.*
- *The R-convex hull of S is the smallest R-convex set that contains S.*
- *For all $R' \leqslant R$, $C_{R'}(S) \subset C_R(S)$,* $\lim_{R\to\infty} C_R(S) = C_\infty(S) = H(S)$.
- *For all $S' \subset S$, $C_R(S') \subset C_R(S)$.*

The R-convex hull of a sample

Definition (R-convex hull estimator)

Let $\mathbb{X}_n$ be a random sample with density f and support S. The R-convex hull of the sample is defined by

$$\widehat{S}_n = C_R(\mathbb{X}_n),$$

Heuristic justification: *If S is regular enough, $C_R(S) \simeq S$ so that a natural estimator of S is $\widehat{S}_n$.*

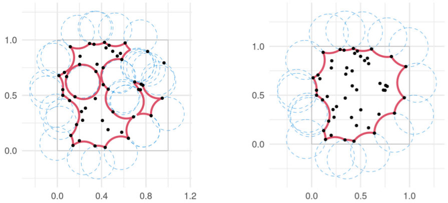


Figure 2: In solid red line the r-convex hull of a sample of 100 uniformly distributed random vectors in $[0,1]^2$. At left $r = 0.15$, while at right $r = 0.2$.

- $\widehat{S}_n$ is the smallest R-convex set containing $\mathbb{X}_n$.
- Its computation of is totally data-driven (alphahull package in R for $d = 2$; no efficient algorithm in higher dimension.)

The Hausdorff distance as a measure of convergence between sets

The Hausdorff distance measures the proximity of two compact sets E and F of $\mathbb{R}^d$.

$$d_H(E,F) = \max\left(\sup_{a\in E} \underline{\mathsf{d}}\,(a,F), \sup_{b\in F} \underline{\mathsf{d}}\,(b,E)\right),$$

where for each $x \in \mathbb{R}^d$ $\underline{\mathsf{d}}\,(x,E) = \min_{y\in E}\|x-y\|^2$, and $\|\cdot\|$ is a norm on $\mathbb{R}^d$.
Problem: We ask that $d_H(S,\widehat{S}_n) \to 0$, but this convergence is not sufficient in general to ensure that $\widehat{S}_n$ performs well.

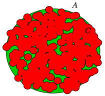


Figure 1.7: *In green $A = B(0,1)$. In red a set C, visually close to A. Although both sets are close in terms of the Hausdorff distance, their shapes are quite different.*

Here, $d_H(A,C)$ is close to zero, even though both sets are not similar. If we are concern not only about the proximity of the sets, but also about the shape similarity, we should ask the estimator to satisfy both

$$d_H(S,\widehat{S}_n) \to 0,\ d_H(\partial S,\partial\widehat{S}_n) \to 0 \qquad \partial S = \text{boundary of } S.$$

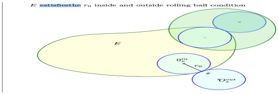
Previous asymptotic results for the R-convex hull of an i.i.d. sample (1)

Definition (Free rolling ball conditions)

- *A compact set $S \subset \mathbb{R}^d$ is said to satisfy the outer λ-rolling condition if for each boundary point $s \in \partial S$ there exists an $x \in S^c$ such that $B(x, \lambda) \cap \partial S = \{s\}$.*
- *A compact set S is said to satisfy the inner λ-rolling condition if S^c satisfies the outer λ-rolling condition.*

If S satisfies the inner (resp. outer) rolling ball condition, we say that **a ball of radius $\lambda > 0$ rolls freely inside (resp. outside)** S.

NB: R-convexity entails the outer R-rolling ball condition.



Assumptions

- *(A1) "close to uniform" hypothesis : f is bounded (in S) from below by a positive constant.*
- *(A2) Free rolling ball condition: A ball of radius $\lambda > 0$ rolls freely inside S and S^c, for all $0 \leqslant \lambda < R$.*

Previous asymptotic results for the R-convex hull of an i.i.d. sample (2)

Theorem (Theorem 3 of Rodríguez-Casal (2007))

If S is a nonempty compact and that (A1) and (A2) are satisfied, then with probability one,

$$d_H(S, \widehat{S}_n) = O\left(\left(\frac{\log n}{n}\right)^{\frac{2}{d+1}}\right), \quad d_H(\partial S, \partial \widehat{S}_n) = O\left(\left(\frac{\log n}{n}\right)^{\frac{2}{d+1}}\right).$$

Therefore, although the family of R-convex sets is much wider than the family of convex sets, $\widehat{S}_n$ achieves the same convergence rate as the convex hull of $\mathbb{X}_n$ in the convex case.

Functionals of the R-convex hull may also be used to estimate related quantities such as the boundary, the volume (Arias-Castro & al., 2016, Baldin, 2017) and the perimeter (Aaron & al., 2022) of S.

Previous asymptotic results for the R-convex hull of an i.i.d. sample

Comments on the assumptions:

(A1) prevents the set S from having holes and being too spiky.

(A2) implies that S and S^c are R-convex. It allows S and its boundary ∂S to be sufficiently smooth. Note that, by merely assuming R-convexity, we cannot ensure that the boundary of the set is smooth. If S is only R-convex the convergence occurs at the rate $1/d$ for $\widehat{S}_n$.

A new result (1)

Hereafter, we relax conditions (A1) and (A2) to more general ones

Assumptions

- *(A'1) : The density f satisfies*

$$\exists\, f_0 > 0,\, \alpha \geqslant 0,\;\, f(x) \geqslant f_0\, \underline{d}\,(x, \partial S)^{\alpha}, \quad \forall x \in S.$$

  This condition ensures that f is regularly enough to ensure a non-sharp boundary. The special case $\alpha = 0$ corresponds to (A1).
- *(A'2) : S fulfills the λ_o-outer and the λ_i-inner rolling ball conditions.*

- Notation : We denote by $|S|_d$ and $|\partial S|_{d-1}$ the d-dimensional volume (volume) and the perimeter of S ($(d-1)$-dimensional volume of ∂S).

A new result (2)

Theorem 1 (Theorem 1 of Aaron, Doukhan, Reboul, 2024)

Let $\mathbb{X}_n$ be an i.i.d. $n-$sample with density f and non empty compact support S. Assume that f satisfies (A'1) and S satisfies (A'2) for some $\lambda_o, \lambda_i > 0$. Let $R < \lambda_o$. Thus, there exists a constant $A(d, \alpha, f_0, R, \lambda_i)$ such that for n large enough, one has with probability 1

$$d_H(\partial\widehat{S}_n, \partial S) = O(\varepsilon_n), \qquad d_H(\widehat{S}_n, S) = O(\varepsilon_n), \tag{1}$$

$$\left||\partial\widehat{S}_n|_{d-1} - |\partial S|_{d-1}\right| = O(\varepsilon_n), \quad \left||\widehat{S}_n|_d - |S|_d\right| = O(\varepsilon_n), \tag{2}$$

with

$$\varepsilon_n = A(d, \alpha, f_0, R, \lambda_i)\left(\frac{\ln n}{n}\right)^{\frac{2}{d+1+2\alpha}} .$$

Theorem 1 highlights that

- As α goes to infinity, the rate becomes worse (the average number of observations near the boundary decreases).
- Moreover, the result highlight the different roles of the outside and inside radius λ_o and λ_i: R must be less than λ_o in order to achieve the convergence and the convergence depends on λ_i via the constant.

Settings

Hereafter we provide convergence rates that generalize to a dependent framework the previous works on the R-convex hull. We consider several contexts of dependence

- Negatively associated sequences
- m-dependent sequences
- Strongly mixing sequences
- θ-weakly dependent sequences.

Convergence rates for the R-convex hull of negatively associated or m-dependent sequences

- Negative association: A finite family of random variables $\mathbb{X}_n$ is said to be negatively associated if, for any two disjoint nonempty subsets A_1 and A_2 of $\{1, 2, \ldots, n\}$, the following inequality holds:

$$\text{cov}(f_1(X_i,\, i \in A_1), f_2(X_j,\, j \in A_2)) \leqslant 0,$$

  where f_1 and f_2 are any two coordinatewise nondecreasing or nonincreasing functions, whenever the covariance is finite.

- m-dependence: A strictly stationary process $\mathbf{X}$ is said to be m-dependent, $m \in \mathbb{N}^*$, if for each t, $(X_s)_{s \leqslant t}$ and $(X_s)_{s > t+m}$.

Theorem 2 (Proposition 3 of Aaron, Doukhan, Reboul, 2024)

Let $\mathbb{X}_n$ be a n-sample of a negatively associated or $m-$dependent sequence $\mathbf{X} = (X_t)_{t \in \mathbb{Z}}$ with marginal density f and non empty compact support S. Assume that f satisfies (A'1) and S satisfies (A'2) for some $\lambda_o, \lambda_i > 0$. Let $R < \lambda_o$.Then the conclusions of Theorem 1 hold.

Strong mixing (Rosenblatt, 1956)

Let $\mathbf{X}$ be a strictly stationary process. Set

$$\mathcal{P} = \sigma(X_t, t \leqslant 0), \quad \mathcal{F} = \sigma(X_t, t \geqslant r).$$

$$\alpha(r) = \sup_{A \in \mathcal{P}, B \in \mathcal{F}} |\mathbb{P}(A \cap B) - \mathbb{P}(A)\mathbb{P}(B)|.$$

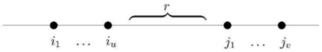
X is strongly mixing $\iff \alpha(r) \to 0$ as $r \to +\infty$.

- Ex : ARMA models with continuous innovations.

θ-dependence (Doukhan & Louhichi, 1999)

$$P = X_{i_1}, \ldots, X_{i_u},\ F = X_{j_1}, \ldots, X_{j_v},\ j_1 - i_u = r$$

$$A \in \sigma(P), B \in \sigma(F),\ \ \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \Leftrightarrow \mathsf{cov}(f(P), g(F)) = 0,\ \|f\|_\infty \leqslant 1, \|g\|_\infty \leqslant 1.$$



Def : Lipschitz modulus of $g : \mathbb{R}^v \to \mathbb{R}$:

$$Lip(g) = \sup_{x \neq y} \frac{|g(x) - g(y)|}{\|x - y\|_1},\ \|x\|_1 = \sum_{i=1}^{v} |x_i|.$$

X is θ-weakly dependent $\Longleftrightarrow |cov(f(P), g(F))| \leqslant vLip(g)\theta(r)$,

$\theta(r) \to 0$ as $r \to +\infty$.

$$\|f\|_\infty \leqslant 1,\ \|g\|_\infty \leqslant 1,\ Lip(g) < \infty.$$

- Ex : ARMA models with discrete innovations.

Convergence rates for the R-convex hull of strongly mixing or θ-weakly dependent sequences

Theorem 3 (Theorem 2 of Aaron, Doukhan, Reboul, 2024)

Let $\mathbb{X}_n$ be be a $n-$sample of a strictly stationary process $\mathbf{X}$ with marginal density f and non empty compact support S. Assume that f satisfies (A'1) and S satisfies (A'2) for some for some $\lambda_o, \lambda_i > 0$. Let $R < \lambda_o$. Assume that $\mathbf{X}$ is strongly mixing or θ-weakly dependent.

- *If the sequence of dependence coefficients of $\mathbf{X}$ has an* ***exponential decay*** *rate $\alpha(q) \leqslant Bb^{-q}$, or $\theta(q) \leqslant Bb^{-q}$, for some $b < 1$,* ***Equations* (1) *and* (2)** *of Theorem 1 are satisfied with*

$$\varepsilon_n = A(d, \alpha, f_0, R, \lambda_i, b) \left(\frac{\ln^2 n}{n} \right)^{\frac{2}{d+1+2\alpha}},$$

  where the constant $A(d, \alpha, f_0, R, \lambda_i, b)$ is explicit.

- *If the sequence of dependence coefficients of $\mathbf{X}$ has a* ***polynomial decay*** *rate $\alpha(q) \leqslant Bq^{-b}$ or $\theta(q) \leqslant Bq^{-b}$ with $b > b_0$,* ***Equations* (1) *and* (2)** *of Theorem 1 are satisfied with*

$$\varepsilon_n = \left(\frac{\ln^a n}{n^{1-\gamma}} \right)^{\frac{2}{d+1+2\alpha}}.$$

  Explicit values of b_0, a and γ are given in each case (α-mixing and θ-dependence).

Convergence rates for the R-convex hull of strongly mixing or θ-weakly dependent sequences

Theorem 3 highlights that

- In every cases, as α goes to infinity, the rate becomes worse.
- In the exponential decay case, the estimation error achieves, up to an additional power of $\ln n$, the same rate than in the independent setting whatever the dependence structure is (strong mixing or θ-weak dependence). The dependence structure only impacts constants.
- In the polynomial decay case, the loss in convergence rate with respect to the independent case is of the order of an additional power of n. Moreover, contrary to the exponential decay case, both the error and the loss depend on the dependence structure.

Key arguments for the proofs of Theorems

Geometric and deterministic arguments (Corollary 1 of Aaron, Doukhan, Reboul, 2024) allow to prove that

1. If S satisfies (A'2), $R \in (0, R_0)$ and there exists a sequence (ε_n) such that

$$\sum_{n=1}^{\infty} p_n < \infty, \quad \text{with} \quad p_n = \mathbb{P}\Big(\exists\, O,\ \underline{\mathbf{d}}(O,S) \leqslant R-\varepsilon_n \text{ and } B(O,R)\cap\mathbb{X}_n = \emptyset\Big), \quad \text{then (1) obtains.}$$

2. Moreover, if $d_H(\mathbb{X}_n, S) \to 0$ and (A'1) holds then (2) obtains.

The proof of both items above reduces in bounding terms of the form

$$\mathbb{P}\left(E_{i,n} \cap \mathbb{X}_n = \emptyset\right), \tag{3}$$

where $E_{i,n}$ consist in balls or intersections of balls.

For instance, for the first item, we can show that

$$p_n \leqslant \sum_{i=1}^{\nu_n} \mathbb{P}\left(E_{i,n} \cap \mathbb{X}_n = \emptyset\right), \quad E_{i,n} = B\left(O_i, R - \frac{\varepsilon_n}{\ln n}\right), \ \forall n > 0, i \in [1, \nu_n]$$

Finding a upper bound for (3) is the main argument of the proofs and depends on the dependence structure of the underlying process $\mathbf{X}$.

Key arguments for the proofs of Theorems

- **Independence setting**
  Let E be a measurable subset of $\mathbb{R}^d$ with $\mathbb{P}(X \in E) = p > 0$. If $\mathbb{X}_n$ is an i.i.d. sample, then

$$\mathbb{P}(\mathbb{X}_n \cap E = \emptyset) = (1-p)^n. \tag{4}$$

  If $\mathbb{X}$ is a negatively associated or $m-$dependent or is a geometrically ergodic Markov chain, Equation (4) still holds (Proposition 3 of the paper).

- **Dependence setting**
  Relaxing the independence assumption leads to a weaker version of Equation (4). Indeed, let $\mathbb{X}_n$ is an n-sample from a strictly stationary process $\mathbf{X}$, let E be a measurable subset of $\mathbb{R}^d$ with $\mathbb{P}(X_t \in E) = p > 0$, we have explicit values of $\mathcal{E}_E(q)$ such that for each $q \in \mathbb{N}$ with $1 \leq q \leq n$ we have

$$\mathbb{P}(\mathbb{X}_n \cap E = \emptyset) \leq (1-p)^{\frac{n}{q}} + \frac{\mathcal{E}_E(q)}{p}, \tag{5}$$

  for several dependence structures and for the cases of sets E of interest.

For the proof of item 1, these inequalities allow to reduce the problem of bounding $\mathbb{P}\left(E_{i,n} \cap \mathbb{X}_n = \emptyset\right)$ to bound $\mathbb{P}(E_{i,n})$ uniformly with respect to $i \in [1, \nu_n]$, which is done using $(A'1)$ and $(A'2)$. We thus can exhibit a sequence (ε_n) such that $\sum_{n=1}^{\infty} p_n < \infty$.

Key arguments for the proofs of Theorems

Proposition (Proposition 1 of Aaron, Doukhan, Reboul, 2024)

Let $\mathbf{X} = (X_t)_{t\in\mathbb{Z}}$ *be a stationary strongly mixing sequence with mixing coefficients* $(\alpha(q))_{q>0}$. *Let* $E \subset \mathbb{R}^d$ *be an arbitrary measurable set such that* $\mathbb{P}(X_t \in E) = p > 0$. *Then* (5) *holds for each* $1 \leq q \leq n$ *with* $\mathcal{E}_E(q) = \alpha(q)$.

Proposition (Proposition 2 of Aaron, Doukhan, Reboul, 2024)

Let $\mathbf{X} = (X_t)_{t\in\mathbb{Z}}$ *be a stationary* $\theta-$*weakly dependent sequence with dependence coefficients* $(\theta(q))_{q>0}$. *We set* ω_d *the volume of* $B(0,1) \subset \mathbb{R}^d$ *and* σ_d *the surface of its boundary.*

1. *For* $E = B(x,r)$, *then there exists* $c > 0$ *such that* (5) *holds with*

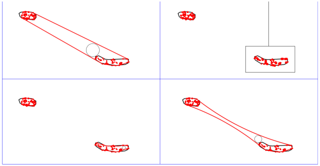
$$\mathcal{E}_E(q) \leqslant \inf_{\varepsilon\in(0,r)} \left(\frac{\theta(q)}{\varepsilon} + c\omega_d 2^d \varepsilon r^{d-1}\right). \tag{6}$$

2. *For* $E = B(x,r_x) \cap B(y,r_y)$ *with* $r_x + r_y - \|x-y\| = \ell$, *and* $\ell \leqslant r_x \wedge r_y$, *then there exists* $c > 0$ *such that* (5) *holds with*

$$\mathcal{E}_E(q) \leqslant \inf_{\varepsilon\in(0,\ell/2)} \left(\frac{2\theta(q)}{\varepsilon} + c\mathcal{V}(r_x,r_y,\ell,\varepsilon)\right), \mathcal{V}(r_x,r_y,\ell,\varepsilon) \underset{\ell\to 0}{\sim} 4c\omega_{d-1}\varepsilon \left(\frac{\ell 2 r_x r_y}{r_x+r_y}\right)^{\frac{d-1}{2}}. \tag{7}$$

Perspectives : data-driven selection of R

Despite of its good properties, $\widehat{S}_n$ presents an important disadvantage in practice: it depends on the commonly unknown parameter R. The influence of R can be considerable.
Example of invasive species in Azorean islands (Rodriguez-Casal & al., 2022). Estimation of its extent of occurrence (EOO). EOO is one of the most widely handled concepts in natural reserve network designs . The International Union for the Conservation of Nature establishes the EOO as a key measure of extinction risk.



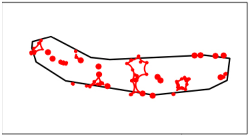


Fig. 2: sample of 740 locations in two Azorean islands. In the first row, $H(\mathbb{X}_{740})$ (left) and $C_{0.03}(\mathbb{X}_{740})$ (right). In the second row, $C_{0.3}(\mathbb{X}_{740})$ (left) and $C_5(\mathbb{X}_{740})$ (right)

Arbitrary choices of R may provide incongruous EOO estimations: small values of R provide fragmented estimators (many isolated points and connected components (first row, right). For intermediate value of R, a realistic reconstruction of the EOO is obtained since sea areas are not inside the estimator (second row, left). However, if large values of R are considered, then $C_r(\mathbb{X}_n)$ basically coincides with $H(\mathbb{X}_n)$ (second row, right).

In the i.i.d. case, Rodríguez-Casal & al. (2016) and Rodriguez-Casal & al. (2022) propose methods of selection of R, which cannot be easily transposed to a dependent setting.

- **C. Aaron, P. Doukhan and L., Reboul. Set estimation under dependence. arXiv: Statistics Theory, 2024.**
- C. Aaron and O. Bodart. Local convex hull support and boundary estimation. Journal of Multivariate Analysis, 147:82-101, 2016.
- C. Aaron, A. Cholaquidis, and R. Fraiman. Estimation of surface area. Electronic Journal of Statistics, 16(2) :3751-3788, 2022.
- E. Arias-Castro, B. Pateiro-López, and A. Rodríguez-Casal. Minimax estimation of the volume of a set with smooth boundary. arXiv: Statistics Theory, 2016.
- N. Baldin. The wrapping hull and a unified framework for estimating the volume of a body. arXiv: Statistics Theory, 2017.
- A. Cholaquidis, Fraiman, and M. Hernández-Banadik. Home-range estimation under a restricted sample scheme. Journal of Nonparametric Statistics, 31:1-20, 2023.
- A. Cholaquidis, R. Fraiman, G. Lugosi, and B. Pateiro-López. Set estimation from reflected brownian motion. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 78, 2014.
- A. Cuevas and A. Rodríguez-Casal. On boundary estimation. Advances in Applied Probability, 36(2):340-354, 2004.
- L. Devroye and G. L. Wise. Detection of abnormal behavior via nonparametric estimation of the support. SIAM Journal on Applied Mathematics, 38(3):480-488, 1980.
- L. Dümbgen, G. Walther. Rates of convergence for random approximations of convex sets, Adv. Appl. Probab. 28, 384-393, 1996
- W. M. Getz and C. C. Wilmers. A local nearest-neighbor convex-hull construction of home ranges and utilization distributions. Ecography, 27:489-505, 2004.
- W. Hardle, B.U. Park, and A.B. Tsybakov. Estimation of non-sharp support boundaries. Journal of Multivariate Analysis, 55(2):205-218, 1995.
- S. Kallel and S. Louhichi. Topological reconstruction of compact supports of dependent stationary random variables. arXiv:2307.11674v1[math.PR], 2023.
- A. Rodríguez Casal. Set estimation under convexity type assumptions. Annales De l‚ÄôInstitut Henri Poincaré: Probabilités et Statistiques, 43:763-774, 2007.
- A. Rodríguez-Casal and P. Saavedra-Nieves. A data-adaptive method for estimating density level sets under shape conditions. The Annals of Statistics, 50(3):1653-1668, 2022.
- A. Rodríguez-Casal and P. Saavedra-Nieves. Spatial distribution of invasive species: an extent of occurrence approach. TEST, 31: 416-441, 2022.
- G. Walther. Granulometric smoothing. The Annals of Statistics, 25(6):2273-2299, 1997.