Optimal dividend strategies for a catastrophe insurer

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(joint with P. Azcue† and N. Muler)

Outline

- Dividend problem in insurance
  - Rich source for stochastic control problems
  - Non-standard problem formulations

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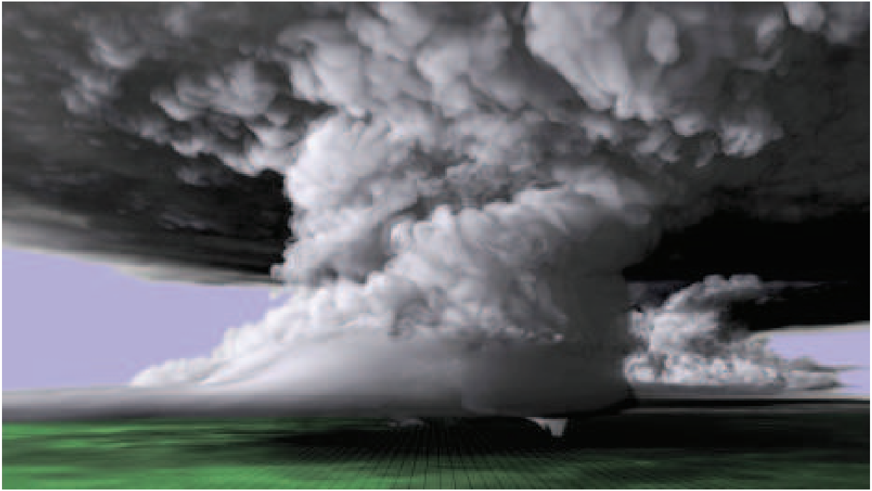
  - Rich source for stochastic control problems
  - Non-standard problem formulations
- Shot-noise Cox process for claim arrivals

  $\hookrightarrow$ Motivation from **Catastrophe Insurance**
- Solve two-dimensional control problem
- Numerical Illustrations / Interpretations

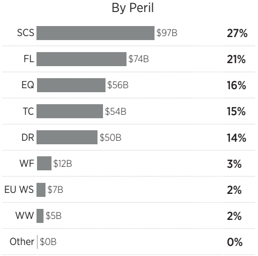
Natural catastrophes: Severe convective storms

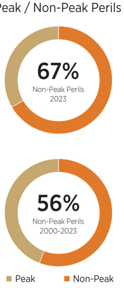


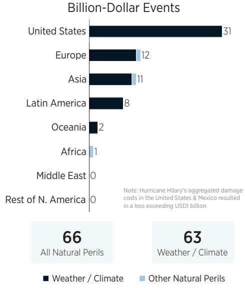
Simulated tornado-producing supercell (30m resolution, EF5 near El Reno and Oklahoma City, 2011)

Orf et al. (2017)

Natural catastrophes: Economic Losses 2023







EQ: Earthquake
DR: Drought
SCS: Severe Convective Storm
EU WS: European Windstorm
TC: Tropical Cyclone
WF: Wildfire
WW: Winter Weather
FL: Flooding

Gallagher Re: NatCat & Climate Report 2023

Natural catastrophes: Comparison 2023 (in bn. USD)

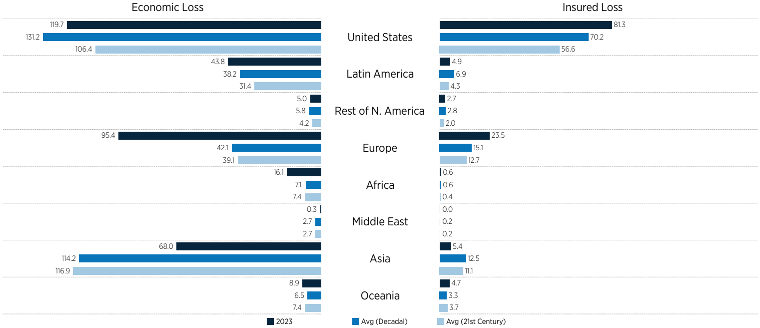


Figure 9: Regional economic and insured losses in 2023 versus recent history | **Data and Graphic:** Gallagher Re

Gallagher Re: NatCat & Climate Report 2023

- Climate Change: reinsurance premiums go up!

Natural catastrophes: Comparison 2023 (in bn. USD)

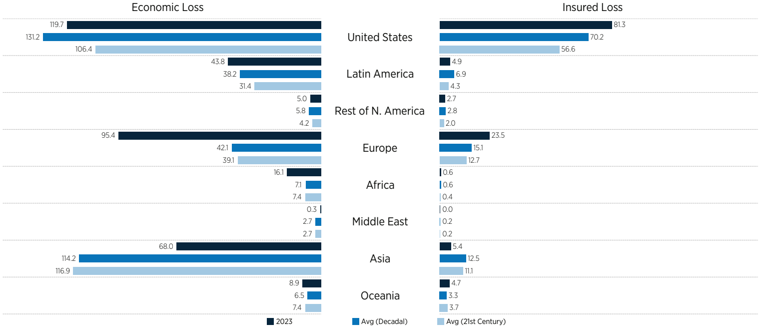
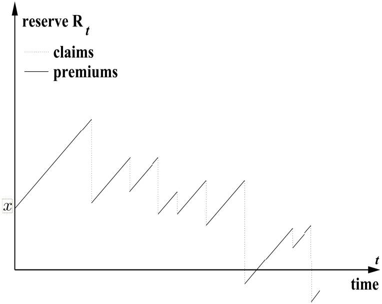


Figure 9: Regional economic and insured losses in 2023 versus recent history | **Data and Graphic:** Gallagher Re

Gallagher Re: NatCat & Climate Report 2023

- Climate Change: reinsurance premiums go up!
- **How to make natural catastrophe insurance attractive?**

Alternative (profitability) criterion:



B. de Finetti (1957)

Expected sum of discounted dividend payments until ruin

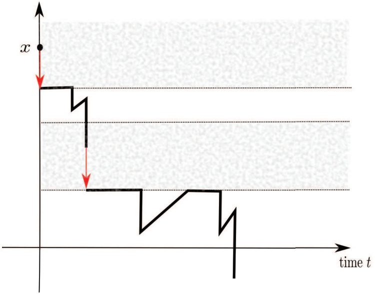
$$\mathbb{E}_x\Big(\int_0^{\tau} e^{-\delta t}\, dD_t\Big) \longrightarrow \text{ maximize!}$$

Stochastic Control Problem:

- Compound Poisson: e.g. Gerber (1969), Schmidli (2008), Azcue & Muler (2014)
- Diffusion: e.g. Gerber (1972), Jeanblanc-Piqué & Shiryaev (1994), Asmussen & Taksar (1997)

e.g. A. & Thonhauser (2009), Avanzi (2009)

Optimal Strategy: Band Strategy



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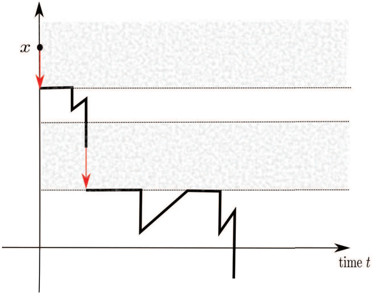
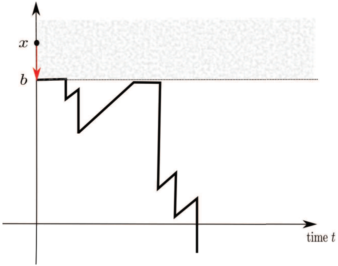
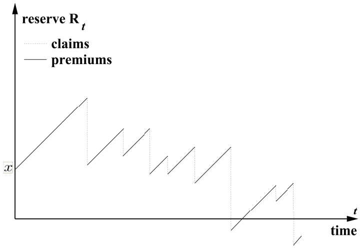
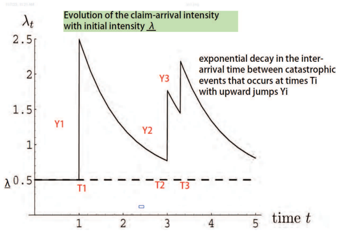


Diffusion, certain compound Poisson/Lévy processes: Barrier Strategy



A compound Cox model





$N(t)\ldots$ **shot-noise Cox process**: $R_t = x + p\,t - \sum_{n=1}^{N(t)} X_n$

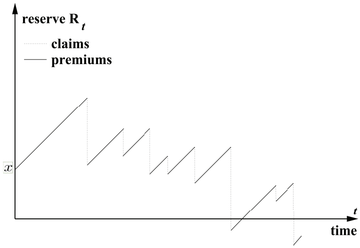
$$\lambda_t = \lambda_t^c + \sum_{k=1}^{\widetilde{N}_t} Y_k \; e^{-d(t-T_k)}$$

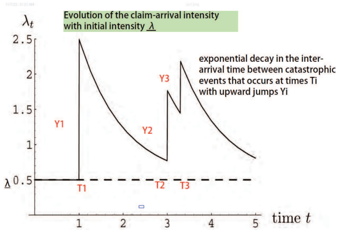
with $\lambda_t^c = \underline{\lambda} + e^{-dt}\,(\lambda - \underline{\lambda})$, $d > 0$

$\widetilde{N}_t = \#\{k : T_k \le t\}\ldots$ Poisson process of constant intensity β

Dassios & Jang (2003), A. & Asmussen (2006), Pojer & Thonhauser (2023)

A compound Cox model





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Premium rate:

$p = (1+\eta)\,\mathbb{E}(X_n)\,\lim_{t\to\infty} \mathbb{E}(\Lambda_t/t) = (1+\eta)\,\mathbb{E}(X_n)(\underline{\lambda} + \beta\mathbb{E}(Y_k)/d)$

The optimal dividend problem

$\Pi_{x,\lambda}$: set of admissible dividend strategies $L = (L_t)_{t\geq 0}$: non-decreasing, càdlàg, adapted, $L_0 \geq 0$ and $L_t \leq R_t$ for $t < \tau^L = \inf\{t \geq 0 : R_t - L_{t^-} < 0\}$.

Two-dimensional stochastic control problem

$$V(x,\lambda) = \sup_{L \in \Pi_{x,\lambda}} \mathbb{E}\left(\int_{0^-}^{\tau^L} e^{-\delta t} dL_t\right)$$

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Two-dimensional stochastic control problem

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Basic properties:

- $V(x,\lambda)$ non-decreasing in x, non-increasing in λ
- $V(x,\lambda)$ locally Lipschitz in x and in $\lambda > \underline{\lambda}$
- $V(x,\lambda) \leq v^{\underline{\lambda}}(x) \leq x + p/\delta$
- $\lim_{\lambda\to\infty} V(x,\lambda) = x$

Hamilton-Jacobi-Bellman equation

$$\max\{\mathcal{L}(V)(x,\lambda), 1 - V_x(x,\lambda)\} = 0,\ x \geq 0,\ \lambda \geq \underline{\lambda} \qquad (\otimes)$$

where

$$\begin{aligned} \mathcal{L}(V)(x,\lambda) = \quad & pV_x(x,\lambda) - d\left(\lambda - \underline{\lambda}\right) V_\lambda(x,\lambda) - (\delta + \lambda + \beta) V(x,\lambda) \\ & + \lambda \int_0^x V(x-y,\lambda) dF_X(y) + \beta \int_0^\infty V(x,\lambda+z) dF_Y(z). \end{aligned}$$

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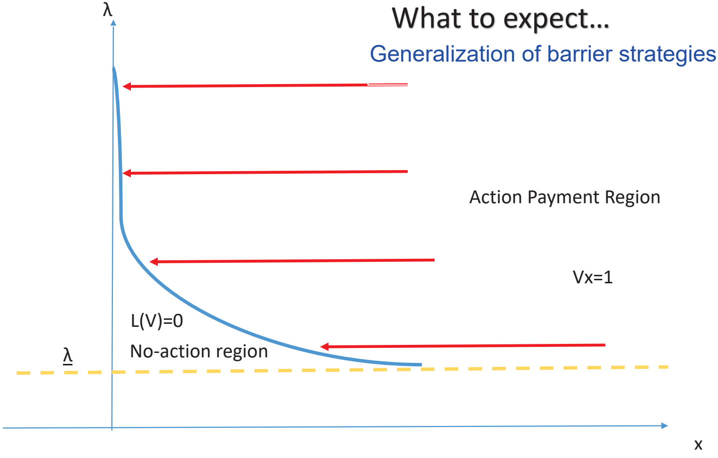
Theorem: The optimal value function V is the smallest viscosity solution of $(\otimes)$ in $[0,\infty) \times [\underline{\lambda},\infty)$ satisfying linear boundedness in x and $\lim_{\lambda\to\infty} V(x,\lambda) = x$.

- Verification Theorem
- $V(x,\lambda) = x - p/\delta + V(p/\delta,\lambda)$ for $x \geq p/\delta \quad \forall \lambda > \underline{\lambda}$

What to expect...

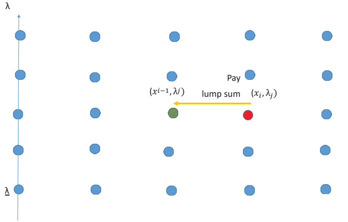
Generalization of barrier strategies



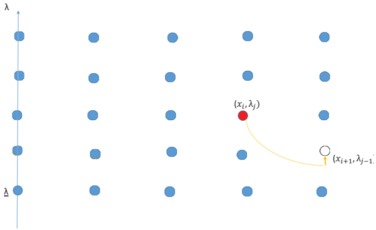
Free boundary problem - numerically very hard!

- **Approximate** by optimal strategies on fine two-dim. grid
- Uniform convergence of these strategies to V

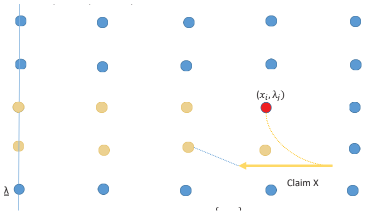
Finite grid



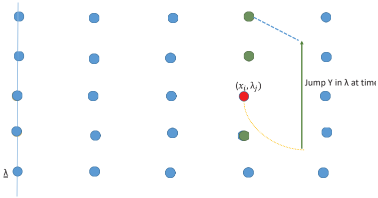
lump sum payment



no dividends until next grid point



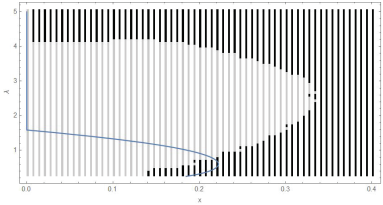
no dividends until next claim



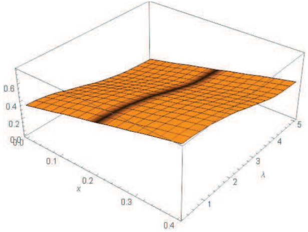
no dividends until next intensity jump

Example 1: $X \sim Exp(10)$, $\underline{\lambda} = 1/4$, $Y \sim Exp(1/2)$, $\beta = 1/2$, $d = 7/10$

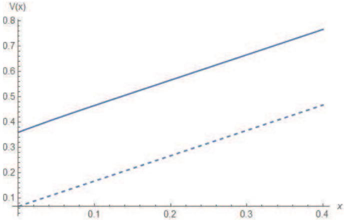
(halftime of intensity increase is 1 year), $\delta = 0.2$, $\eta = 0.2$ ($\lambda_{\text{av}} = 1.679$, so $p = 0.201$).



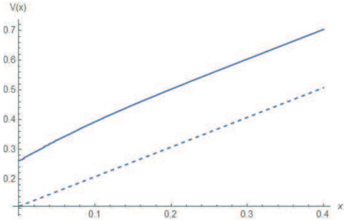
Action regions (dark) and non-action regions (light) of the optimal strategy as a function of λ and x



$V(x, \lambda)$



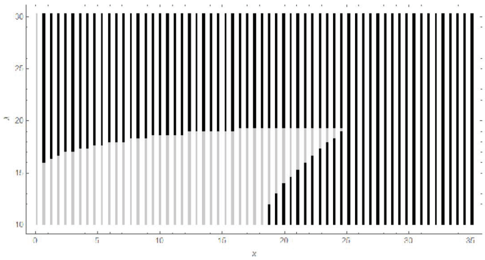
$V(x, \underline{\lambda})$ (solid) vs. $V_{\text{CL}}(x, \underline{\lambda})$ (dashed)



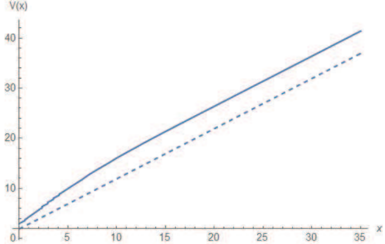
$V(x, \lambda_{\text{av}})$ (solid) vs. $V_{\text{CL}}(x, \lambda_{\text{av}})$ (dashed)

Example 2: $X \sim Erl(2,1)$, $\underline{\lambda} = 10$, $Y \sim Exp(1/2)$, $\beta = 0.2$, $d = 0.2$, $\delta = 0.1$, $\eta = 0.07$ ($\lambda_{av} = 12$, so $p = 25.68$).

hom. Poisson: Optimal 2-band!

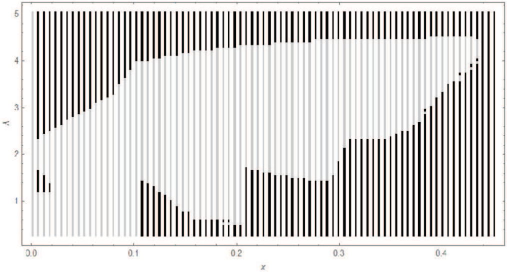


Action regions (dark) and non-action regions (light) of the optimal strategy as a function of λ and x



$V(x, \lambda_{av})$ (solid) vs. $V_{CL}(x, \lambda_{av})$ (dashed)

Example 3: Like Example 1, but with deterministic claim sizes: $F_X(x) = I_{[1/10,\infty)}$.



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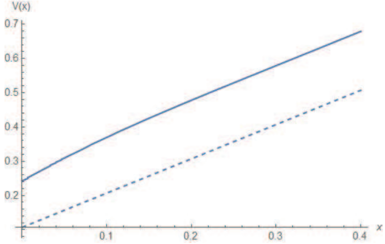

$V(x, \lambda_{\text{av}})$ (solid) vs. $V_{\text{CL}}(x, \lambda_{\text{av}})$ (dashed)

Conclusion

- New two-dimensional stochastic control problem
- Variability of claim intensity can be advantage!
- Potential for catastrophe insurance (IFRS, regulation?)

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Possible Extensions:

- Unknown intensity: filtering
- Non-stationarities (climate change!)
- Hawkes processes (cyber risk) etc.

Thank you for your attention!

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