Powers correlation analysis of non-stationary assets

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Outline

Constant unconditional variance assumption may be unrealistic:

- Stărică and Granger (2005): found strong evidence of non constant variance for large samples of daily stock returns.
- Stărică (2003): For volatility forecasts of stock returns.
- Engle and Rangel (2008), Hafner and Linton (2010): The spline GARCH.
- Subba Rao (2006), Kokoszka and Leipus (2000): ARCH(∞) models allowing unconditional non constant variance.
- ... many others ...

Constant unconditional variance assumption may be unrealistic:

Diagnostic tools:

- Numerous tests for detecting non constant unconditional variance: e.g. Berkes, Horváth and Kokoszka (2004) in a GARCH context.
- Test for second order dynamics in presence of a non-constant variance: Patilea and Raïssi (2014).

Non-constant daily zero returns probability

Illiquid stocks=Let us say presence of daily zero returns

- Time-varying illiquidity levels are often not taken into account in the financial econometric literature
- Relatively small companies in all markets.
- Lesmond (2005): very common in emerging markets.
- So, let us review some representative examples taken from the Chilean stock market...!!

Empirical facts: Returns with possibly non constant zero returns probability

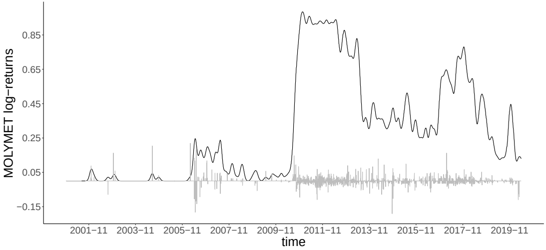

Figure: The daily log-returns of the Molymet stock. Data source: Yahoo Finance.

Capital increase of more than 216 millions Dollars, announced during the extraordinary shareholders meeting by August 13th, 2010.

Empirical facts: Returns with possibly non constant zero returns probability

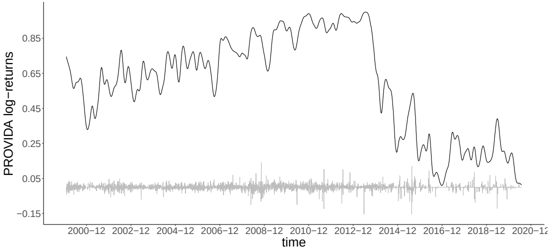

Figure: The daily log-returns of the Provida stock. Data source: Yahoo Finance.

Take-over bid of Metlife on the pension funds administration company Provida during September 2013. On that occasion Metlife acquired more than 90% of the share capital of Provida.

Empirical facts: Returns with possibly non constant zero returns probability

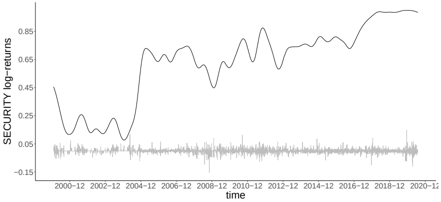

Figure: The daily log-returns of the Security stock. Data source: Yahoo Finance.

- Merger by absorption of the Dresdner Bank Lateinamerika in September 2004.
- Issued more than 32.8 millions new stocks after the capital increase announced during the extraordinary shareholders meeting by December 29th, 2004.

Empirical facts: Returns with possibly non constant zero returns probability

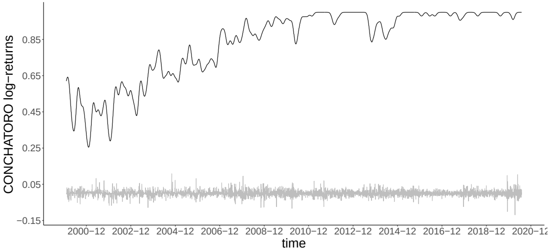

Figure: The daily log-returns of the Conchatoro stock. Data source: Yahoo Finance.

Increase of the liquidity due to the fast developing of the emerging Chilean stock market in the 2000's.

Empirical facts: Returns with possibly non constant zero returns probability

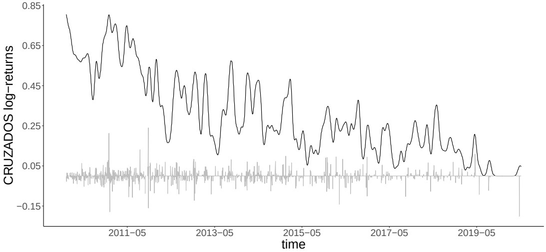

Figure: The daily log-returns of the Cruzados stock. Data source: Yahoo Finance.

Long-run decrease of the liquidity.

Empirical facts: Returns with possibly constant zero returns probability

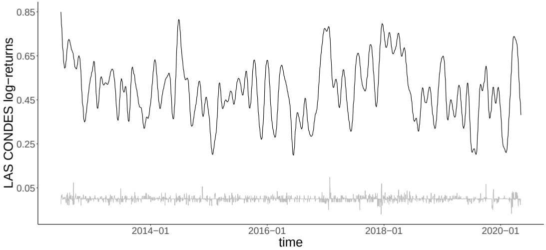

Figure: The daily log-returns of the Clinica Las Condes stock. Data source: Yahoo Finance.

Sometimes the zero returns probability may be assumed constant.

Empirical facts: Intraday data with non-constant zero returns probability

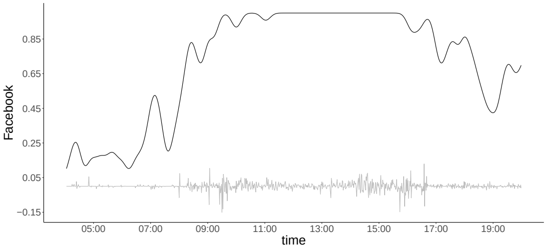

Figure: The 1-minute log-returns of the Facebook stock. Data source: Firstdata.

Zero returns can be always observed for small enough time periods.

Our goal

Test higher-order correlations in the returns dynamics.

- Inhomogeneous non zero returns distribution over time can be confused with long-run/long memory volatility effects.
- Correct tools to assess short run effects or evaluate the shock persistency.

Pre-publication of this work: arXiv:2104.04472v1.

The framework

- Let $r_1, \dots, r_n$ be the observed returns, with n the sample size.
- Consider the binary process $(a_t) \subset \{0, 1\}$.
- $r_t = a_t \widetilde{r}_t$, with $\widetilde{r}_t$ partially unobserved.
- $\widetilde{r}_t = \sigma_t \eta_t$, where σ_t deterministic if no higher-order correlations.

$r_t = 0$ may be explained by a variety of facts: such as trading costs, or rounded prices....

The framework

Investigate the power returns serial correlations

$$\widehat{\Gamma}_0^{(\delta)}(m) := \left(\hat{\rho}_0^{(\delta)}(1), \ldots, \hat{\rho}_0^{(\delta)}(m)\right)', \text{with} \hat{\rho}_0^{(\delta)}(h) := \hat{\gamma}_0^{(\delta)}(h)\hat{\gamma}_0^{(\delta)}(0)^{-1},$$

where $\hat{\gamma}_0^{(\delta)}(h) = n^{-1}\sum_{t=1+h}^{n}\left(|\widetilde{r}_t|^\delta - \bar{\widetilde{r}}^{(\delta)}\right)\left(|\widetilde{r}_{t-h}|^\delta - \bar{\widetilde{r}}^{(\delta)}\right)$, and $\bar{\widetilde{r}}^{(\delta)} = n^{-1}\sum_{t=1}^{n}|\widetilde{r}_t|^\delta$.

Problem: $\widetilde{r}_t$ is not fully observed!

The framework

We investigate the behavior of the feasible statistic

$$\widehat{\Gamma}_s^{(\delta)}(m) := \left(\hat{\rho}_s^{(\delta)}(1), \ldots, \hat{\rho}_s^{(\delta)}(m)\right)', \text{with} \hat{\rho}_s^{(\delta)}(h) := \hat{\gamma}_s^{(\delta)}(h)\hat{\gamma}_s^{(\delta)}(0)^{-1},$$

where $\hat{\gamma}_s^{(\delta)}(h) = n^{-1} \sum_{t=1+h}^{n} \left(|r_t|^\delta - \bar{r}^{(\delta)}\right) \left(|r_{t-h}|^\delta - \bar{r}^{(\delta)}\right)$ and $\bar{r}^{(\delta)} = n^{-1} \sum_{t=1}^{n} |r_t|^\delta$.

The framework

Studied cases:

Case 1: Constant variance and probability.

Case 2: Constant variance, time-varying probability.

Case 3: Time-varying probability and variance.

The results

Case 1: Everything is ok with the classical higher-order serial correlations

Proposition

Let $\delta > 0$, and suppose that $\sigma_t > 0$ is constant, and $0 < P(a_t = 1) < 1$ is constant. Then, under additional assumptions, for any integer $m \geq 1$, $\sqrt{n}\widehat{\Gamma}_s^{(\delta)}(m) \xrightarrow{d} \mathcal{N}(0, I_m)$.

The classical powers correlations may be used safely...

The results

Case 2: The problem

Proposition

Let $\delta > 0$, and suppose that $\sigma_t > 0$ is constant (no higher order dynamics). Moreover, assume that $0 < P(a_t = 1) < 1$ is not constant. Then, under additional assumptions for any integer $m \geq 1$, $\widehat{\Gamma}_s^{(\delta)}(m) \xrightarrow{a.s.} C_{0,a} \in \mathbb{R}^m$, and all the components of the vector $C_{0,a}$ are equal and strictly positive.

Using the classical powers correlations is not a good idea...

The results

Case 2: Correct higher order serial correlations:

$$\widehat{\Gamma}_{ns}^{(\delta)}(m) := \left(\hat{\rho}_{ns}^{(\delta)}(1), \ldots, \hat{\rho}_{ns}^{(\delta)}(m)\right)', \text{ with } \hat{\rho}_{ns}^{(\delta)}(h) := \hat{\gamma}_{ns}^{(\delta)}(h)\hat{\gamma}_{ns}^{(\delta)}(0)^{-1},$$

where

$$\hat{\gamma}_{ns}^{(\delta)}(h) = n^{-1}\sum_{t=1+h}^{n}\left(|r_t|^{\delta} - \bar{r}^{(\delta)}\frac{P(a_t = 1)}{\bar{a}}\right)\left(|r_{t-h}|^{\delta} - \bar{r}^{(\delta)}\frac{P(a_{t-h} = 1)}{\bar{a}}\right),$$

and $\bar{a} = n^{-1}\sum_{t=1}^{n} P(a_t = 1)$.

The results

Case 2: Correct the higher-order serial correlations.

Proposition

Assume that $\sigma_t > 0$ is constant. Suppose that $0 < P(a_t = 1) < 1$ is not constant. Then, under additional assumptions, for any integer $m \geq 1$, we have $\sqrt{n}\widehat{\Gamma}_{ns}^{(\delta)}(m) \xrightarrow{d} \mathcal{N}(0, \varsigma I_m)$, as $n \to \infty$, where ς is given in the paper.

The results

Case 3: The problem

Proposition

Let $\delta > 0$ and suppose that σ_t is deterministic non-constant, and $P(a_t = 1)$ is time-varying. Then, under additional assumptions, for any integer $m \geq 1$, $\widehat{\Gamma}_s^{(\delta)}(m) \xrightarrow{a.s.} C_{0,\sigma} \in \mathbb{R}^m$. If $v^\delta(\cdot)g(\cdot)$ is a constant function, then $C_{0,\sigma}$ is the null vector, otherwise all the components of the vector $C_{0,\sigma}$ are equal and strictly positive.

Again, using the classical powers correlations is not a good idea...

The results

Case 3: Correct the higher-order serial correlations.

$$\widehat{\Gamma}^{(\delta)}_{ns,\sigma}(m) := \left(\hat{\rho}^{(\delta)}_{ns,\sigma}(1), \ldots, \hat{\rho}^{(\delta)}_{ns,\sigma}(m)\right)',$$

with $\hat{\rho}^{(\delta)}_{ns,\sigma}(h) := \hat{\gamma}^{(\delta)}_{ns,\sigma}(h)\hat{\gamma}^{(\delta)}_{ns,\sigma}(0)^{-1}$, where

$$\hat{\gamma}^{(\delta)}_{ns,\sigma}(h) = n^{-1} \sum_{t=1+h}^{n} \left\{|r_t|^\delta - E\left(|r_t|^\delta\right)\right\} \left\{|r_{t-h}|^\delta - E\left(|r_t|^\delta\right)\right\}.$$

The results

Case 3: Correct the higher-order serial correlations.

Proposition

Let $\delta > 0$, and suppose that $\sigma_t > 0$ is deterministic non-constant and $0 < P(a_t = 1) < 1$ is time-varying. Then, under additional assumptions, for any integer $m \geq 1$, we have $\sqrt{n}\widehat{\Gamma}^{(\delta)}_{ns,\sigma}(m) \xrightarrow{d} \mathcal{N}(0, \zeta I_m)$, where ζ is given in the paper.

Feasible statistics

- In the above results $E\left(|r_t|^{\delta}\right)$ and $P(a_t = 1)$ are assumed known.
- We need to estimate them to build feasible statistics.

Feasible statistics

- Time-varying probability:

$$\widehat{P(a_t = 1)} = \sum_{j=1}^{n} w_{tj}(b_a) a_j,$$

- Time-varying δ moment of (r_t):

$$\widehat{E(|r_t|^\delta)} = \sum_{j=1}^{n} w_{tj}(b_\tau) |r_j|^\delta,$$

- Smoothing weights:

$$w_{tj}(b) = (nb)^{-1} K\left((t-j)/(nb)\right).$$

Feasible statistics

- The Kernel fulfill standard conditions.
- The bandwidths b_a and b_τ are taken in the range $\mathcal{B}_n = [c_{min} b_n, c_{max} b_n]$ with $0 < c_{min} < c_{max} < \infty$ and $nb_n^4 + 1/nb_n^{2+\gamma} \to 0$ as $n \to \infty$, for some $\gamma > 0$.

Feasible statistics $\widetilde{\Gamma}_{ns}^{(\delta)}(m)$ and $\widetilde{\Gamma}_{ns,\sigma}^{(\delta)}(m)$ can be obtained by plugin the estimators defined above.

Feasible tests

Proposition

Under suitable conditions, we have

$$\sqrt{n}\left|\widehat{\Gamma}_{ns}^{(\delta)}(m) - \widetilde{\Gamma}_{ns}^{(\delta)}(m)\right| \xrightarrow{p} 0,$$

uniformly with respect to $b_a \in \mathcal{B}_n$, and

$$\sqrt{n}\left|\widehat{\Gamma}_{ns,\sigma}^{(\delta)}(m) - \widetilde{\Gamma}_{ns,\sigma}^{(\delta)}(m)\right| \xrightarrow{p} 0,$$

uniformly with respect to $b_\tau \in \mathcal{B}_n$.

Practical issues

- The tools are implemented using a bootstrap procedure ($B = 3999$ replications).
- The bandwidths are selected using a leave-one-out cross validation criterion (LOOCV).
- $H_0^{(\delta)}$ vs. $H_1^{(\delta)}$ are tested with

$$H_0^{(\delta)} \text{ no power correlations of order } \delta.$$

- ACF of power returns are built.
- $\delta = 1$ is taken in all our experiments (Taylor (1986) effect).

Notations

- **Classical:** the usual serial autocorrelations of powers returns.
- **RP:** autocorrelations of powers returns robust to time-varying zero returns probability.
- **RPV:** autocorrelations of powers returns robust to both time-varying zero returns probability and variance.

Simulated processes

- Under $H_0^{(\delta)}$: iid (r_t) (Case 1)
- Under $H_0^{(\delta)}$: Non constant zero returns probability (Case 2)
- Under $H_0^{(\delta)}$: Non constant unconditional variance and zero returns probability (Case 3)

Table: The frequencies (in %) of adaptive and classical autocorrelations outside their respective nominal 95% confidence bands, obtained from $R = 5000$ independent replications, Case 1 under $H_0^{(\delta)}$.

	lags	1	2	3	4	5	20	40	60
Classical	$n = 100$	4.12	4.20	4.20	4.66	4.40	2.30	1.14	0.28
	$n = 200$	4.52	4.46	4.64	4.12	4.76	3.78	2.70	1.68
	$n = 400$	5.36	4.30	4.56	4.90	4.56	4.22	3.36	2.80
	$n = 800$	5.20	5.04	4.70	4.64	4.90	5.02	4.22	4.36
RP	$n = 100$	6.04	6.16	5.98	6.70	6.48	5.14	4.56	4.08
	$n = 200$	6.06	5.80	5.64	5.62	6.16	5.36	5.48	4.98
	$n = 400$	5.76	4.96	5.82	5.34	5.62	5.08	5.14	4.82
	$n = 800$	5.18	5.12	4.98	4.94	5.54	6.08	5.14	5.88
RPV	$n = 100$	6.24	6.28	6.34	7.14	6.76	5.18	4.54	3.84
	$n = 200$	6.04	6.10	5.88	5.60	6.52	5.58	5.82	5.02
	$n = 400$	6.04	5.00	5.78	5.38	5.80	5.20	5.30	5.00
	$n = 800$	5.42	5.24	5.06	5.02	5.58	6.04	5.30	6.00

Table: The same as above but for Case 2 under $H_0^{(\delta)}$.

	lags	1	2	3	4	5	20	40	60
Classical	$n = 100$	46.04	44.44	42.30	40.50	40.68	8.64	0.64	1.38
	$n = 200$	71.68	70.12	71.44	70.04	69.76	53.64	17.60	1.52
	$n = 400$	94.70	94.64	94.74	94.54	93.66	91.90	83.10	67.36
	$n = 800$	99.88	99.80	99.90	99.92	99.88	99.7	99.8	99.3
RP	$n = 100$	6.20	5.78	5.62	6.16	5.04	4.14	3.44	1.82
	$n = 200$	5.88	6.24	5.92	6.04	5.64	4.66	4.66	5.40
	$n = 400$	5.70	5.36	5.24	5.28	5.70	5.02	4.88	4.60
	$n = 800$	5.58	5.50	5.26	5.64	5.32	6.06	5.28	5.80
RPV	$n = 100$	6.70	6.36	7.10	7.86	6.34	4.42	3.42	1.78
	$n = 200$	6.70	6.98	6.96	6.60	6.50	4.72	4.52	5.44
	$n = 400$	6.12	6.34	5.92	6.20	6.20	5.26	5.04	4.48
	$n = 800$	5.86	5.60	5.70	5.80	6.10	6.44	5.54	5.68

Table: The same as above but for Case 3 under $H_0^{(\delta)}$.

	lags	1	2	3	4	5	20	40	60
Classical	$n = 100$	79.84	78.88	78.16	74.56	74.68	20.46	0.56	5.58
	$n = 200$	97.38	97.80	97.62	97.36	97.64	90.44	46.00	1.52
	$n = 400$	99.98	99.96	100.00	100.00	100.00	99.96	99.72	97.06
	$n = 800$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
RP	$n = 100$	4.86	3.98	4.30	4.46	4.30	3.76	3.24	3.22
	$n = 200$	4.44	4.40	4.56	4.28	4.40	4.36	3.84	4.78
	$n = 400$	5.44	4.90	4.84	4.84	4.76	4.90	4.74	4.28
	$n = 800$	6.86	6.48	6.38	6.24	6.48	6.74	6.20	6.00
RPV	$n = 100$	6.36	6.24	6.78	7.38	6.30	4.28	2.58	1.10
	$n = 200$	5.88	6.46	6.16	6.06	6.34	4.68	4.38	4.98
	$n = 400$	5.32	5.58	5.50	5.98	5.76	5.26	5.02	4.20
	$n = 800$	5.40	5.52	5.74	5.42	5.54	6.22	5.26	5.64

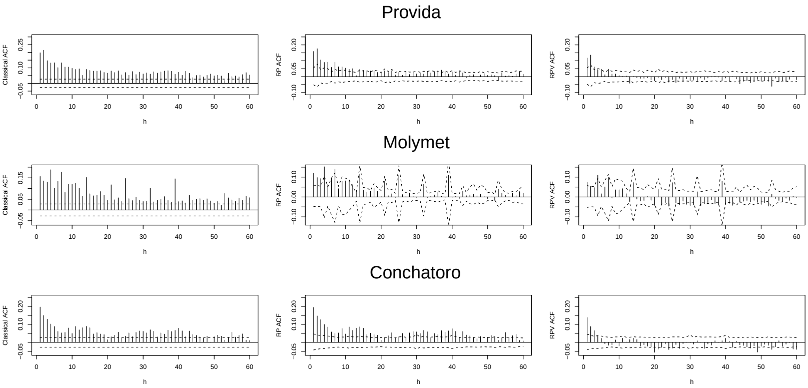

Figure: The classical (left column), RPV and RP (middle and right columns) absolute returns autocorrelations ($\delta = 1$) for $h = 1, \ldots, 60$. The dashed lines correspond to the bootstrap and classical 95% confidence bands.

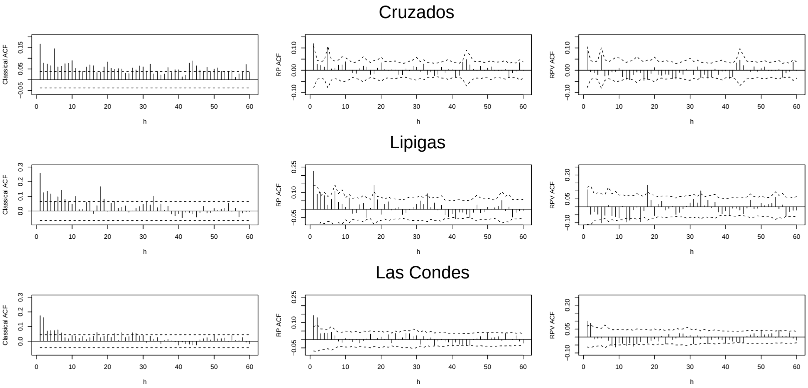

Figure: The same as above.

Conclusion

- Non constant unconditional zero returns probability is a common feature.

In this framework the standard portmanteau test is

- Unable to distinguish between non constant liquidity levels and second order residual autocorrelation.

Adaptive portmanteau test which

- Control the type I errors reasonably well
- Able to detect second order dynamics

⇒ Help for the volatility specification when the unconditional variance and liquidity levels are not constant.