Some ideas about modeling real situations using hypoelliptic SDE equations with or without boundary conditions

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Abstract

Newton's second law writes very simply: $ma = F$, where m is the mass of the mobile, a is its acceleration and F is the force acting on the mobile. If we denote by $x(t) \in \mathbb{R}^d$ the trajectory that the mobile follows and if we assume that the force is written $-\gamma(x, x')x' - \nabla V(x) + dW$, being γ the friction force coefficient, $V : \mathbb{R}^d \to \mathbb{R}$ the potential whose gradient is the force exerted on the mobile and W a standard Brownian motion (BM) in $\mathbb{R}^d$, Newton's second law gives us the equation for the nonlinear harmonic oscillator forcing with a white noise

$$mx''(t) + \gamma(x(t), x'(t))x'(t) + \nabla V(x(t)) = dW(t).$$

We interpret this equation as a second-order stochastic differential equation (SDE). We can also describe the motion in phase space by putting $X(t) = x(t)$ e $Y(t) = x'(t)$ we have the following SDE system

$$\begin{aligned} dX(t) &= Y(t)dt \\ mdY(t) &= -\gamma(X(t), Y(t))Y(t) - \nabla V(X(t)) + dW(t). \end{aligned}$$

In this conference, inspired by the seminal works of Brillinger et al. we consider the solution of the system, the existence of an invariant measure, and the exponential mixing of the solution. However, the mobile will move restricted to bounded domains but only in the space variable. We then dwell on applications of such models: description of animal movement in bounded domains, and diffusion of pollutants in closed water bodies etc. Our emphasis will be on dealing in a mobile restricted to a bounded region $\Omega \subset \mathbb{R}^d$.

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