RATE OF CONVERGENCE IN THE STRONG LAW OF LARGE NUMBERS FOR RECORDS

O. I. KLESOV, O. V. KOLESNIK

Consider a sequence of independent identically distributed random variables $\{X_k\}$ and the corresponding sequence of *record times*:

$$T_1 = 1, \qquad T_k = \min\{j : \ X_j > X_{T_{k-1}}\}, \quad k > 1.$$

The random variables X_{T_k}, $k \geqslant 1$, are called *records*. The

$$\mu_k = \text{number of records up to moment } k$$

is of interest in this talk. The strong law of large numbers for $\{\mu_k\}$ tells us that

$$\lim_{n\to\infty} \frac{\mu_n}{\ln(n)} = 1 \qquad \text{almost surely.}$$

An improvement of this statement could be called the Marcinkiewicz–Zygmund strong law of large numbers:

$$\lim_{n\to\infty} \frac{\mu_n - \ln(n)}{(\ln(n))^{1/r}} = 0 \qquad \text{almost surely}$$

for any $1 \leqslant r < 2$. We study a similar result in a more general setting called F^α-scheme.

Let X be a random variable and F_X be its distribution function being nondegenerate in the sense that

$$P(X = 0) \neq 0.$$

Let $\{\alpha_k\}$ be a sequence of nonnegative numbers. A sequence of independent random variables $\{X_k\}$ is called the F^α-scheme associated with X and $\{\alpha_k\}$ if

$$F_{X_k} = F_X^{\alpha_k}.$$

Define T_k and μ_k, $k \geqslant 1$, in the same way as above. Then one can prove for the F^α-scheme that

$$\lim_{n\to\infty} \left[\frac{\mu_n}{\ln(A_n)} - \frac{1}{A_n} \sum_{k=1}^{n} \frac{\alpha_k}{A_k} \right] = 0 \qquad \text{almost surely,}$$

The first author is supported in part by DFG.

where $A_n = \alpha_1 + \cdots + \alpha_n$ (see [2]). We propose a method for finding the sequences $\{f(n)\}$ and $\{g(n)\}$ such that

$$\lim_{n\to\infty}\left[\frac{\mu_n}{g(n)} - f(n)\right] = 0 \qquad \text{almost surely.}$$

Based on this result, we are able to solve the same problem for record times $\{T_k\}$. In doing so we use the idea of pseudo-regularly functions, since T_n is an generalized renewal process constructed from μ_n (see [1]).

Литература

[1] V. V. Buldygin, K.-H. Indlekofer, O. I. Klesov, J. G. Steinebach (2018). *Pseudo-Regularly Varying Functions and Generalized Renewal Processes*. Springer, Cham, 482+XXIIpp.

[2] P. Doukhan, O. I. Klesov, J. G. Steinebach (2015) *Strong laws of large numbers in an F^α-scheme* in book *Festschrift dedicated to 65^{th} birthday of Prof. Paul Deheuvels*, Springer, Cham, 287–303

National Technical University of Ukraine "Igor Sikorsky Kyiv Politechnic Institute, Kyiv, Ukraine

Email address: voselk@gmail.com