where the supremum is taken over all pairs of partition \((A_i, \ldots, A_j)\) and \((B_i, \ldots, B_j)\) of a set \(S\) such that \(A_i \in \mathcal{P}_t\) for each \(i\) and \(B_j \in \mathcal{F}_m\) for each \(j\).

We say that \(X\) is absolutely regular if \(\lim_{m \to \infty} \beta(m) = 0\).

**CFG estimator of absolutely regular sequences of bivariate extremes**

Let \(Z = (Z_t, Z_u)\) be defined from \(X\) by:

\[
Z_t = \frac{\log P(X_t)}{\log P(X_t) + \log P(X_u)}, \quad Z_u = \frac{\log P(X_u)}{\log P(X_t) + \log P(X_u)}
\]

and denote by \(H_t\) and \(H_u\) its cdf. Then, it can be shown that (Zhang et al. 2008)

\[
\log \Lambda(t) = \int_0^t H_1(z) \, dz - \int_0^t H_2(z) \, dz - \int_0^t H(z) \, dz + \int_0^t H_1(z) H_2(z) \, dz - \int_0^t H(z) \, dz
\]

This naturally suggests the definition of the CFG estimator of \(\Lambda\) based on a sample \((X_1, \ldots, X_n)\) of \(X\):\n
- **Definition:** Let \(Z = (Z_t, Z_u)\) be a sample of \(X\). The CFG estimator of \(\Lambda\) is defined by the weighted estimator

\[
\hat{\Lambda}_n(s) = \left( \frac{\hat{A}_n(s)}{\hat{A}_n(s)} \right)^{1/n}, \quad s \in [0, 1], \quad \tilde{A}_n(l) = 1,
\]

where \(\Lambda\) is an appropriately chosen nonnegative weight function in \([0, 1]\) and \(\hat{H}_1\) and \(\hat{H}_2\) are the empirical counterparts of \(H_1\) and \(H_2\).

**Empirical version of the CFG:** The calculation of \(\Lambda\) relies on the knowledge of \(X\) marginals through \(H_i\). However, marginals are rarely known so that we can replace \(H_i\) by their empirical counterparts (1). All the results below hold for this version.

- **Asymptotic properties:** In the sequel we denote by \(D_1[0, 1]\) the usual D space on \([0, 1]\) with Skorokhod topology. Let \(\mathcal{B}^*\) the bivariate centered Gaussian process with covariance function

\[
\Gamma(s, t) = \int_0^1 \int_0^1 \mathbb{E} \left[ (\mathcal{B}(s) - \Lambda(s)) (\mathcal{B}(t) - \Lambda(t)) \right] \, ds \, dt,
\]

with \(s_1 = s, s_2 = s, t_1 = t, t_2 = t\). Let \(\Lambda^*\) be \(\mathcal{B}^*\) centered Gaussian process. Then \(\mathcal{B}^*\) and \(\Lambda^*\) are two independent processes.

**Theorem:** Let \((X_1, \ldots, X_n)\) be a strictly stationary absolutely regular sequence of extremes with sequence of coefficients of absolutely regularity \((\alpha_1, \ldots, \alpha_n)\). Suppose that \(\Lambda\) has a bounded first derivative and that \(\Lambda\) is a bounded function on \([0, 1]\). Then

1. If \(\beta(m) = O(m^{-1})\) for some \(\beta > 1 / \sqrt{7}\) then \(\lim_{m \to \infty} \hat{\Lambda}_n(s) = \Lambda(s) \Delta_s 0\)

2. If \(\beta(m) = O(m^{-1})\) for some \(\beta > 1 / 2\) then \(\lim_{m \to \infty} \hat{\Lambda}_n(s) = \Lambda(s) \Delta_s 0\), with \(\gamma(s) < \infty\)

3. If \(\beta(m) = O(m^{-1})\) for some \(\beta > 1 / 2\) then \(\lim_{m \to \infty} \hat{\Lambda}_n(s) = \Lambda(s) \Delta_s 0\), with \(\gamma(s) < \infty\)

**References**


**References**
