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On Periodic Generalized Poisson INGARCH models: Application to COVID data

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INTRODUCTION

- A generalization of the Generalized Poisson INGARCH (1,1) model, [1] to the periodic case.
- The model has a periodic autocorrelation structure.
- The periodic GPINGARCH, can account for both overdispersion and underdispersion.
- The basic probabilistic and statistical properties of this models are fully studied.
- Application on Covid-19 infections cases in Germany is considered.

MATERIALS AND METHODS

* A periodically correlated Integer-Valued process $\{X_t, t \in \mathbb{Z}\}$, with period S (where S ≥2), is said to satisfy a Periodic $GPINGARCH_S(1,1)$, if it the following form:

$$ {}^{X_t}/_{\mathcal{F}_{t-1}} \backsim GP(\lambda_t^*, \kappa_t), \quad {}^{\lambda_t^*}/_{(1-\kappa_t)} = \lambda_t = \alpha_{0,t} + \alpha_{1,t} X_{t-1} + \beta_t \lambda_{t-1}, $$

where the parameters $\alpha_{0,t} > 0$, $\alpha_{1,t} \geq 0$, $\beta_t \geq 0$ and $max(-1, -\lambda_t^*/4) < \kappa_t < 1$, are periodic in t with period S, and $\mathcal{F}_{t-1}$ denotes, as usually, the -σ-field generated by the past of the process.

* The closed form of the mean $\mu_{X,s}$ for the periodically stationary $PGPINGARCH_S(1,1)$ model in the mean, under the condition, is given by

$$ \prod_{s=1}^{S} (\alpha_{1,s} + \beta_s) < 1, $$

$$ \mu_{X,s} = \left[1 - \prod_{i=1}^{S} (\alpha_{1,i} + \beta_i)\right]^{-1} \sum_{j=0}^{S-1} \left[\prod_{i=1}^{S} (\alpha_{1,i} + \beta_i)\right] \alpha_{0,s-j} \,. $$

* The closed form of the variance $\gamma_X^{(s)}(0)$ for the periodically stationary $PGPINGARCH_S(1,1)$ model in the second order, under the condition, is given by

$$ \prod_{s=1}^{S} (\alpha_{1,s} + \beta_s)^2 < 1, $$

$$ \gamma_X^{(s)}(0) = \varphi_s^2 \mu_{X,s} + \left[1 - \prod_{i=1}^{S} (\alpha_{1,i} + \beta_i)^2\right]^{-1} \sum_{j=0}^{S-1} \left[\prod_{i=1}^{S} (\alpha_{1,i} + \beta_i)^2\right] F_{s-j} \,, $$

where $\varphi_s = {}^{1}/_{(1-\kappa_s)}$ and $F_s = \alpha_{1,s}^2 \varphi_{s-1}^2 \mu_{\lambda,s-1}$.

* The periodic autocovariance $\gamma_X^{(s)}(h)$, $s = 1,2,\ldots,S$ and $h \in \mathbb{N}^*$ of the periodic $GPINGARCH_S(1,1)$ model is given by,

$$ \gamma_X^{(s)}(h) = \begin{cases} (\alpha_{1,s} + \beta_s)\gamma_X^{(s-1)}(0) - \beta_s \varphi_{s-1}^2 \mu_{X,s-1}, & h = 1 \\ \prod_{i=1}^{h} (\alpha_{1,s-i+1} + \beta_{s-i+1})\, \gamma_X^{(s-h)}(0) & h \geq 2 \end{cases} $$

RESULTS

* An application of the periodic $GPINGARCH_S(1,1)$ model for the daily number COVID infection cases in Germany, from July 15 to September 15, 2020 [2,3], is considered.

* The data are overdispersed and exhibits a periodical autocorrelation structure, of a period S=7.

* We consider the maximum likelihood estimators for the parameters.

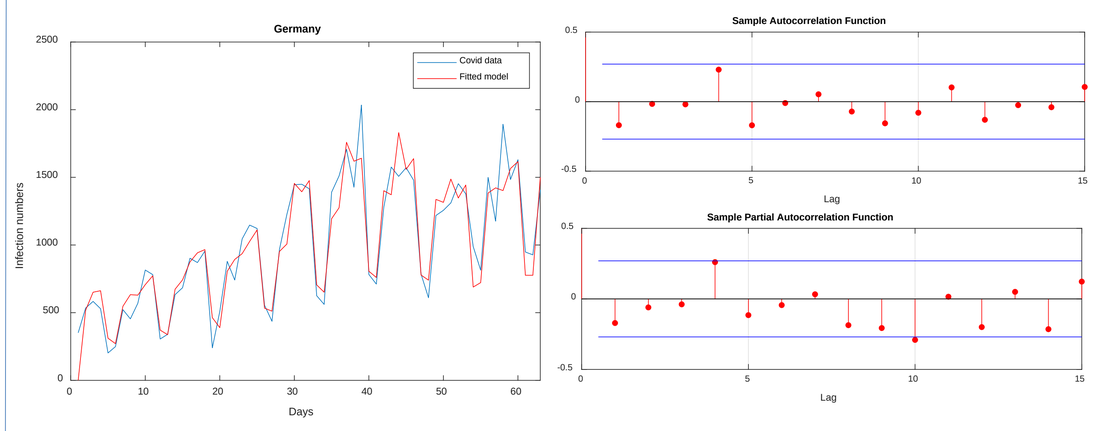

Fig.1. An adjusted trajectory opposed to the Covid data and the residuals correlogram

	R²	Pearson residuals		AIC	BIC
		Mean	Variance		
Germany	**98,77**	**0,0130**	**0,8469**	**641,3579**	**729,3657**

- The residuals correlogram of the $PGPINGARCH_{13}(1,1)$ fitted model do not indicate any statistical significance.
- The adequacy of the fitted model is not statistically rejected.
- The mean and the variance of the Pearson residuals, are close to, 0 and 1 respectively.
- The size of the time series is very small (N=63) compared to the number of parameters to estimate which is 28, therefore the selected model can be improved for a larger size.

CONCLUSION

- The introduced periodic Generalized Poisson INGARCH (1,1) model, can account for both overdispersion and underdispersion and the hidden periodicity in the autocovariance structure frequently encountered in real life;
- The first and the second moment periodically stationary conditions are established, and the closed form expressions of the first two moments are obtained;
- The periodic autocovariance structure is also derived;
- An application on Covid infection cases in Gemany is considered;
- The fitted model shows a good performance in modeling overdispersed and periodic data, although the small size of the time series compared to the number of parameters to estimate

Références
[1] Zhu, F. (2012). Modeling overdispersed or underdispersed count data with generalized Poisson integer-valued GARCH models. Journal of Mathematical Analysis and Applications, 389, 58 - 71.
[2] Doukhan, P., A. Leucht, and M.H. Neumann. (2020). Mixing properties of non-stationary INGARCH(1,1) processes. Working paper Retrieved from https://arxiv.org/abs/2011.05854.
[3] European Centre for Disease Prevention and Control (2020 - 10- 02). https://data.europa.eu/euodp/de/data/dataset/covid-19-coronavirus-data.