FLTs for nested Karlin's occupancy scheme generated by light-tailed distributions

Oleksandr Iksanov*

Let $(p_k)_{k\in\mathbb{N}}$ be a discrete probability distribution for which the counting function $x \mapsto \#\{k \in \mathbb{N} : 1/p_k \le x\}$ belongs to the de Haan class Π (the distribution $(p_k)_{k\in\mathbb{N}}$ is then light-tailed). Consider a deterministic weighted branching process generated by $(p_k)_{k\in\mathbb{N}}$. A nested Karlin's occupancy scheme is the sequence of Karlin's balls-in-boxes schemes in which boxes of the jth level, $j = 1, 2, \ldots$ are identified with the jth generation individuals and the hitting probabilities of boxes are identified with the corresponding weights. The collection of balls is the same for all generations, and each ball starts at the root and moves along the tree of the deterministic weighted branching process according to the following rule: transition from a mother box to a daughter box occurs with probability given by the ratio of the daughter and mother weights.

Assuming there are n balls, denote by $\mathcal{K}_n^{(j)}$ the number of occupied (ever hit) boxes in the jth level. I shall discuss a functional limit theorem for the vector-valued process $(\mathcal{K}^{(1)}_{\lfloor e^{T+u}\rfloor}, \ldots, \mathcal{K}^{(j)}_{\lfloor e^{T+u}\rfloor})_{u\in\mathbb{R}}$, for each $j \in \mathbb{N}$, properly normalized and centered, as $T \to \infty$. The limit is a vector-valued process whose components are independent stationary Gaussian processes. I shall provide an integral representation of the limit process.

A more delicate functional limit theorem will also be given for

$$(\mathcal{K}^{(1)}_{\lfloor e^{T+u}\rfloor}(1), \mathcal{K}^{(1)}_{\lfloor e^{T+u}\rfloor}(2), \ldots, \mathcal{K}^{(1)}_{\lfloor e^{T+u}\rfloor}(i_1); \ldots; \mathcal{K}^{(j)}_{\lfloor e^{T+u}\rfloor}(1), \mathcal{K}^{(j)}_{\lfloor e^{T+u}\rfloor}(2), \ldots, \mathcal{K}^{(j)}_{\lfloor e^{T+u}\rfloor}(i_j))_{u\in\mathbb{R}}$$

with arbitrary $j \in \mathbb{N}$ and arbitrary $i_1, \ldots, i_j \in \mathbb{N}$. Here, $\mathcal{K}_n^{(j)}(i)$ is the number of the jth level boxes containing i balls (out of n).

The talk is based on a joint work [1, 2] with Z. Kabluchko (Münster) and V. Kotelnikova (Kyiv).

References

[1] A. Iksanov, Z. Kabluchko and V. Kotelnikova, *A functional limit theorem for nested Karlin's occupancy scheme generated by discrete Weibull-like distributions.* J. Math. Anal. Appl. **507** (2022), 125798.

*Faculty of Computer Science and Cybernetics, Taras Shevchenko National University of Kyiv, Ukraine; e-mail address: iksan@univ.kiev.ua

[2] A. Iksanov and V. Kotelnikova, *Small counts in nested Karlin's occupancy scheme generated by discrete Weibull-like distributions*. Stoch. Proc. Appl. **153** (2022), 283–320.